On Chen's invariants and their applications

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Abstract

According to Nash's embedding theorem, every Riemannian manifold can be isometrically embedded in some Euclidean spaces with sufficiently high codimension, [1]. It was hoped by this theorem to have an extrinsic help by regarding Riemannian manifolds as Riemannian submanifolds.

On the other hand, the only known necessary Riemannian condition for a general Riemannian manifold to be minimal isometrically immersed in a Euclidean space regardless of codimension was that the Ricci tensor of the manifold satisfies $Ric \leq 0$. For this reason, in [2], Chern proposed to search about another Riemannian obstructions for a Riemannian manifold to be minimal isometrically immersed in a Euclidean space.

Almost three decades ago, Chen invented new types of Riemannian invariants named δ-invariants or the so-called Chen's invariants and constructed general optimal inequalities including these new invariants and the squared mean curvature of Riemannian submanifolds. Thus, he used these invariants to determine an optimal lower bound for the mean curvature of these submanifolds. Submanifolds attaining this bound are said to be ideal submanifolds. Moreover, by means of these invariants, he obtained new obstructions for a Riemannian manifold to be minimal isometrically immersed not only in Euclidean spaces but also in space forms (see for the details [3]). Here, I would like to point out that his invariants and inequalities and so ideal submanifolds have been studied extensively by many geometers so far (for instance, see [3]).

During this talk, I will first introduce Chen's invariants and mention about the obstructions expressed above. Next, I will talk about some applications of these invariants. Finally, as an application of these invariants, I will present some of my recent results joint work with Luc Vrancken given in [4].

References

[1] J. F. Nash, *The imbedding problem for Riemannian manifolds*, Annales of Mathematics, **63** (1956), 20-63.

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[4] H. Yıldırım, L. Vrancken, *$\delta^{\sharp}(2,2)$-ideal centroaffine hypersurfaces of dimension* 5, Taiwanese Journal of Mathematics, **21** (2017), no.2, 283-304.